Exploring the Theory of the Rainbow

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Good afternoon! I am a student of mathematics from the University of Utrecht in the Netherlands, specializing in mathematics education. I also work as a mathematics teacher in a Dutch highschool. As a final project of my master's thesis, I am currently developing a website about the theory of the rainbow to be used by first year university students of mathematics or physics, and by highschool students who are in their final year.

Two years ago our government introduced a new program for the final years of highschool. In this new program, students have to do a project which involves at least two different fields, for example mathematics and physics. The theory of the rainbow is very suitable for a project of this type. It has obvious physical aspects such as reflexion and refraction of light, and it involves not only geometry, but also the derivative of the arcsine in combination with the chain rule.

I have developed two teaching aids that can be used in two ways: either by the teacher to explain the theory, or by the students to explore the theory by themselves.

The rainbow is caused by sunlight falling on a raincloud which consists of a very large number of raindrops. We imagine each raindrop as a very small sphere. We call the straight line joining the centre of the raindrop and the centre of the sun the *axis.* For our purposes, we consider all rays of sunlight to be parallel to this axis. A ray of sunlight which hits the raindrop will be reflected or refracted in the plane through the ray and the centre of the raindrop. In order to study the path of rays of light in the raindrop, one has to intersect the raindrop by an arbitrary plane through the axis.

I have made a computer model of the behaviour of the rays of light in a plane of this kind, using the French program CABRI-géomètre for plane geometry. In this model you can investigate the path of one ray of light; you can, for instance, change the angle between the incident ray and the normal to the raindrop. It is important to note that the primary rainbow results precisely when light is refracted once, reflected once, refracted once again.
By changing the angle between the incident ray and the normal, we see that there will be a maximum angle between the incident rays and the emergent rays. This maximum is really easy to visualize using CABRI. We also see an accumulation around the maximum: it looks as if a beam of parallel rays of light emerges from the raindrop.
By adding a line in the model, parallel to the emergent ray, we can visualize the angle between the incident and the emergent ray, and CABRI can also give us the size of the angle in degrees.
Analytically, the maximal angle can be determined by differentiating a function involving the arcsine. For different colors the maximum angles are slightly different because every color has its own refractive index. In the computer model you can change the refractive index, and thus see the different maximum angles.

In the second teaching aid, we go to three dimensions by doing an experiment with a spherical bottle filled with water, a spotlight (with a parabolic mirror to get a parallel beam of light) and a cardboard screen. We make a hole in the screen the same size as

the spherical part of the bottle. If we aim the spotlight through the hole to the sphere of the bottle, we see a circular rainbow on the screen. This experiment is based on an idea of the late Professor Minnaert (1893-1970) of Utrecht, who used sunlight. A spotlight makes the experiment feasible in a classroom.
We obtain useful information on the path of the rays by interrupting it in three critical places:

1. Place your finger between the spotlight and the bottle. You see a shadow on the screen. If you move your finger down, the shadow moves up.
2. Wet your finger and touch the front of the sphere. If you find the right place, which may be a little hard to do, you will see a (weak) shadow. Again, if you move your finger down (a little in this case), the shadow moves up.
3. Finally, if you place your finger between the bottle and the screen, you will trivially see a shadow.

Then you can let your students compare the two-dimensional computer experiments with the three-dimensional experiments involving the spherical bottle. You can also guide the students with questions and suggestions like:

- In the spherical bottle experiment, if there is no screen, where would you place your eye to see for example red light emerging from the bottle?
- A rain shower is a wall of very small raindrops, and each raindrop is a very small spherical bottle.
- From which raindrops in this “wall” of rain do rays of sunlight reach your eye?

It is now easy to complete the simple theory of the primary rainbow.

For this presentation and more information see the website www.hagmolen.nl.
Thank you for your attention!